THE EFFECTS OF ROTATION ON THE MAIN-SEQUENCE TURNOFF OF INTERMEDIATE-AGE MASSIVE STAR CLUSTERS

Wuming Yang[1,2], Shaolan Bi[1], Xiangcun Meng[2], Zhie Liu[1]

ABSTRACT

The double or extended main-sequence turnoffs (MSTOs) in the color-magnitude diagram (CMD) of intermediate-age massive star clusters in the Large Magellanic Cloud are always interpreted as age spreads. However, it has been confirmed that the age spreads do not exist in young massive star clusters in the Large Magellanic Cloud, which is challenging the scenario of the age spreads. **The result of effects of rotation on the MSTOs of star clusters is conflicting in previous works. Compared with previous works, we consider the effects of rotation on the MS lifetime and take a different efficiency of rotational mixing.** Our calculations show that rotating models have a fainter and redder turnoff with respect to non-rotating counterparts **with ages between about 0.8 and 2.2 Gyr,** but they have a brighter and bluer turnoff when age **is larger than** 2.4 Gyr. The spread of the MSTO caused by a typical rotation rate **is equivalent to the effect of** an age spread of about 200 Myr **in the CMD of** intermediate-age star clusters. Rotation could lead to the double or extended MSTOs in the CMD of the star clusters with **ages between about 0.8 and 2.2 Gyr.** But the extension is not significant, even does not exist in **young clusters. If the efficiency of the mixing were high enough, the effects of the mixing would counteract the effect of the centrifugal support in the late stage of evolutions, the rotationally induced extension would disappear in the old intermediate-age star clusters; and young clusters would have a blueward extended MSTO.**

Subject headings: stars: rotation — stars: evolution — globular clusters: general

1. Introduction

Since Mackey & Broby Nielsen (2007) **and Mackey et al. (2008)** discovered the double main-sequence turnoffs (MSTOs) in the color-magnitude diagram (CMD) of intermediate-age star clusters, such as NGC 1846, 1806, and 1783 in the Large Magellanic Cloud (LMC), the phenomenon of the double or extended MSTOs have been discovered in the more and more star clusters (Mackey et al. 2008; Glatt et al. 2008; Milone et al. 2009; Girardi et al. 2009; Goudfrooij et al. 2009, 2011a; Keller et al. 2012; Piatti 2013). Some star clusters, such as NGC 1751 in the LMC and NGC 419 (Milone et al. 2009; Girardi et al. 2009; Rubele et al. 2011) in the Small Magellanic Cloud, have not only the double or extended MSTOs but two distinct red clumps (RCs). **Recently, Girardi et al. (2013) found the NGC 411 in the Small Magellanic Cloud also has an extended MSTO. Moreover, Omega Centauri exhibits a main-sequence (MS) bifurcation (Piotto et al. 2005), and NGC 2808 (Piotto et al. 2007) and NGC 6752 (Milone et al. 2013) possess a triple MS split.** These discoveries are not consistent with the classical hypothesis that a star cluster is composed of stars belonging to a simple, single stellar population with a uniform

[1]Department of Astronomy, Beijing Normal University, Beijing 100875, China; yangwuming@ynao.ac.cn; yangwuming@bnu.edu.cn

[2]School of Physics and Chemistry, Henan Polytechnic University, Jiaozuo 454000, Henan, China

age and chemical composition.

The double or extended MSTOs have mostly been interpreted as that the population of those star clusters have bimodal age distributions (Mackey & Broby Nielsen 2007; Mackey et al. 2008) or age spreads of about 100 - 500 Myr (Milone et al. 2009; Girardi et al. 2009; Rubele et al. 2010, 2011; Keller et al. 2012; Piatti 2013). However, Platais et al. (2012) noted that this long period of star formation seems to be odds with the fact that none of the younger clusters are known to have such a trait. Bastian & Silva-Villa (2013) found that young massive clusters NGC 1856 and 1866 in the LMC do not have such large age spreads, **which strongly queries the scenario of the age spreads. The age is about 280 Myr for NGC 1856 (Kerber et al. 2007) and $\sim 160-250$ Myr for NGC 1866 (Brocato et al. 2003).** Moreover, the lasting time for explaining the double MSTOs and the dual RCs of the NGC 1751 seem to be not consistent (Yang et al. 2011). The contributions of interactive binary stars to the double MSTOs and dual RCs were studied by Yang et al. (2011). They found that binary interactions and merging can reproduce the dual RCs and extended MSTO in the CMD of an intermediate-age star cluster. However, the fraction of the interactive binary systems is too low to explain the observed properties. Moreover, the binary interactions make the stars have a younger apparent age than non-interactive systems, i.e. young stars are minority, which seems to be contrary to the finding of Milone et al. (2009), who found that about 70% of stars belong to the blue and bright (young) MSTO and around 30% to the red and faint (old) MSTO. The predict of the interactive binaries seems to be contrary to this finding.

The effects of rotation on the structure and evolution of MS stars are mainly in three ways: (1) **The effect of the centrifugal support leads** to a decrease in the effective temperature and luminosity by decreasing the effective gravity; (2) the mixing of elements results in a decrease in the stellar radius and an increase in the effective temperature by changing the distributions of elements; (3) the Von Zeipel effect and the mixing of elements can affect the instability of convection by changing the radiative and adiabatic temperature gradients. If the convective core increases, the lifetime of the MS will be prolonged, vice versa **(Maeder 1987; Talon et al. 1997; Maeder & Meynet 2000; Yang et al. 2013a)**.

The effects of rotation on the MSTO of intermediate-age **star clusters** were first studied by Bastian & de Mink (2009). They obtained that rotating stars have a lower effective temperature and luminosity than non-rotating counterparts and concluded that stellar rotation can mimic the effect of a double population. However, Girardi et al. (2011) calculated the evolution of rotating models by using the Eggenberger et al. (2010) code and obtained that rotating models have a slightly hotter and brighter turnoff with respect to non-rotating ones. They concluded that rotational effect could not explain the presence of the extended MSTO. In Bastian & de Mink (2009) models, the effect of rotation on the age of stars was neglected and the effects of the rotationally induced mixing may be **weaker compared to those in Girardi et al. (2011) models**, which leads to the fact that their rotating models have a lower temperature and luminosity than non-rotating ones in the whole stage of the MS. Moreover, they only calculated the evolution of **the star with a mass of 1.5 $M_{\odot}$,** assuming that the results for the star can be applied to others. In fact, the effects of rotation on the structure and evolution of stars depends on the mass of stars for a given rotation rate **(Maeder & Meynet 2000; Yang et al. 2013a)**. On the contrary, in the models of Girardi et al. (2011), the effects of the mixing may be **more efficient**, which results in the fact that their rotating models have a higher effective temperature than non-rotating ones in the late stage of the MS for stars of any mass. In the early stage of the MS of Eggenberger et al. (2010) models, the rotating models also exhibit a lower effective temperature and luminosity, which mainly results from the **effect of the centrifugal support. The latest evolutionary tracks calculated by Georgy et al. (2013) also show that the rotating models have a lower effective temperature compared to the non-rotating ones in the early stage of the MS (see their Fig. 11).** If the extent of the effect of mixing in the models of Girardi et al. (2011) were different, the results might be changed. Thus the effects of rotation on the MSTO need to be carefully rechecked.

Furthermore, the observed characteristics of ω Centauri and NGC 2808 can be interpreted by that a fraction of globular cluster stars have sizable helium enhancements over the primordial value (Piotto et al. 2005, 2007). In addition, there are abundance anomalies in other globular clusters (Gratton et al. 2004; Milone et al. 2013). In order to understand the self-enrichment in globular cluster stars, Ventura et al. (2001) and D'Antona et al. (2002) suggest that the low-mass stars may have been polluted at the surface by accretion from the gas that was lost from the evolving intermediate-mass asymptotic giant branch stars, which requires a timescale of a few 100 Myr. However, mass shedding models from fast rotating massive stars (Decressin et al. 2007) or massive binaries (De Mink et al. 2009) can enrich the cluster on a timescale of a few Myr. The rotational mixing in massive stars (Hunter et al. 2008; Brott et al. 2011) and solar models (Yang & Bi 2007; Bi et al. 2011) is far from settled. Other mixing processes may exist in the Sun and stars (Basu & Antia 2008; Hunter et al. 2008; Brott et al. 2011), such as magnetic fields, gravity waves, and so on.

The evolutionary tracks of rotating models with different rotation rates were published by several groups (Brott et al. 2011; Georgy et al. 2013). However, their calculations did not cover the mass range between 1.3 and 1.7 $M_\odot$. In this work, we also focus on the effects of rotation on the MSTO of intermediate-age massive **star clusters**. The paper is organized as follow. We simply show our stellar **evolutionary tracks** in section 2. The results are represented in section 3. In section 4, we discuss and summarize the results.

2. Evolutionary tracks

2.1. Evolution code and assumptions

We used the Yale Rotation Evolution Code (Pinsonneault et al. 1989; Yang & Bi 2007) to compute the evolutions of stellar models with and without rotation. The input physics and some initial parameters are the same as **those** used in Yang & Bi (2007) and Yang et al. (2013a,b). The meridional circulation, the Goldreich-Schubert-Fricke instability, and the secular shear instability are considered (Endal & Sofia 1978; Pinsonneault et al. 1989) in all rotating models. The magnetic effects (Yang & Bi 2006) are only considered in stars with a convective envelope, i.e. with mass less than about 1.3 $M_\odot$. In more massive stars, the magnetic effects, magnetic braking, **and mass loss are neglected, i.e., the total angular momentum is assumed to be conserved.** The transport of angular momentum and the mixing of elements are treated as a diffusion process in our models (Pinsonneault et al. 1989; Yang & Bi 2006). **The efficiency of rotational mixing is described by a parameter f_c, which is less than unity and used to account for how the instabilities mix material less efficiently than they transport angular momentum. Usually, the value of f_c is about 0.02-0.03 (Chaboyer & Zahn 1992; Yang & Bi 2006; Hunter et al. 2008; Brott et al. 2011). Here, we adopted a higher value, i.e., 0.2.**

The initial metallicity Z and hydrogen abundance X was fixed at 0.008 and 0.743, respectively. For a given mass, the rotating and non-rotating models share the same initial parameters except for the rotation rate. Rotating models were evolved from the zero-age MS (ZAMS) to the end of the MS, **assuming the initially uniform rotation of 1.0×10^{-4}, 1.5×10^{-4}, and 2.0×10^{-4} radian s^{-1} which corresponds to an initial period of about 0.73, 0.49, and 0.37 day or a rotation rate of about 0.2, 0.3, and 0.4 times Keplerian rotation rate on ZAMS, respectively.** The period of 0.49 day produces a surface velocity of about 130-180 km s^{-1} for stars with masses between 1.3-3.0 $M_\odot$ on ZAMS, whose typical velocity is about 150 km s^{-1} (Royer et al. 2007). **Compared with the initial velocity on the ZAMS of 150 km s^{-1} in Girardi et al. (2011) models, our initial velocity is varing with mass. In the simulation of Bastian & de Mink (2009), the initial rotation rate is around 0.4 times Keplerian rotation rate on the ZAMS, which is higher than ours. The value of f_c is 0.0228 in Bastian & de Mink (2009) models (see Brott et al. (2011)), which is lower than our value of 0.2.**

2.2. Evolutionary tracks

The evolutionary tracks of rotating and non-rotating models with $M = 1.15$ and $2.2\ M_{\odot}$ are shown in Fig. 1. For the star with $M = 1.15\ M_{\odot}$, due to the presence of magnetic braking, the effects of mixing are almost dominant in the whole MS stage. Thus the rotating model mainly exhibits a higher effective temperature than non-rotating one at the same age. **For stars with mass less than 1.3 $M_{\odot}$, magnetic braking leads to the fact that rotation rate decreases rapidly in a few hundred Myr. Thus the effects of rotation on stars are dominated by the rotational mixing, even more sensitive to the efficiency of the mixing than to the rotation rate.** For the star with $M = 2.2\ M_{\odot}$, in the early stage of the MS phase, the **effect of centrifugal support** dominates the influences of rotation on the structure and evolution of the star; therefore the rotating model has a lower effective temperature and luminosity than non-rotating one. However, **as the evolution proceeds, efficient mixing leads to the fact that the outer edge of convective core is slightly extended (see Fig. 2), which enhances the effects of mixing (Maeder & Meynet 2000; Yang et al. 2013a).** The efficient mixing prolongs the lifetime of the MS by feeding fresh hydrogen fuel into the hydrogen-burning region and makes the rotating models exhibit a higher effective temperature than non-rotating one at the same age, which is consistent with the result of Maeder (1987) and Eggenberger et al. (2010).

The evolutionary tracks of models with $M = 1.4$, 1.5, and 1.6 $M_{\odot}$ are shown in Fig. 3, where the positions of the models with **a given age** are labeled by the notation '$\circ$'. In the early stage of the MS, due to the fast rotation and the fact that the gradients of chemical compositions are small, the effect of rotational mixing is not significant and the **effect of the centrifugal support** plays a dominant role, thus the rotating models have a lower luminosity and effective temperature than non-rotating one. But as the evolution proceeds, the rotating models can exhibit a higher luminosity and a lower effective temperature than non-rotating one at the same age. The effective temperature of rotating models also can be higher than that of the non-rotating one, which depends on the age **and rotation rate of stars.**

The **effect of the centrifugal support leads** to a decrease in the effective temperature and gravity of stars. The decrease of the gravity **acts as an effective** decrease in the mass of stars. **The mass** of the convective core of MS stars decreases with decreasing stellar mass. Therefore, the **centrifugal** effect results in a decrease in the convective core (see the Fig. 2). **The simulation of Julien (1996) also shows that convective penetration may be hindered in rotating stars.** Hydrogen is ignited at the same temperature. The smaller the **convective core**, the less the hydrogen that can be consumed in the core, the shorter the lifetime of the MS. Hence the **effect of centrifugal support** leads to an acceleration of the evolution of stars. **For example, the rotating model with $M = 1.5\ M_{\odot}$ and $P = 0.49$ day has evolved to the MS hook at the age of 1.5 Gyr. However, the non-rotating counterpart is still on MS with a higher effective temperature and a lower luminosity (see Fig. 3), which leads to that the rotating model seems to be older than non-rotating one. In fact, they have the same age.**

The rotational mixing can bring hydrogen fuel into the core from outer layers and transport the products of H-burning outwards. The helium in the convective core is brought to the radiative region, which leads to an increase in the density ρ at the bottom of the radiative envelope. The adiabatic gradient ∇_{ad} **is proportional to** $1/\rho$, thus the convective core can be slightly increased by the mixing. The increase bring more hydrogen fuel into the core and enhance the effect of mixing which prolong the lifetime of the MS of stars and lead to an increase in the effective temperature by increasing the mean density of stars.

The effects of rotation on the structure and evolution of stars are a result of the competition between the **effect of the centrifugal support and that** of rotational mixing. For stars with mass larger than 1.3 $M_{\odot}$, due to without magnetic braking, the **effect of the centrifugal support** plays a dominant role in the early stage of the MS. In the late stage of the MS, although the **effect of the centrifugal support** plays an important role, the effects of mixing partly counteract the influences of the **centrifugal** effect, even are dominant. Thus, the convective core of the ro-

tating 1.5 $M_\odot$ model with **the initial rotation period of 0.49 day** is almost as larger as that of non-rotating one in the late stage of the MS (Fig. 2). **In the case of the evolution of rotating model with $M = 1.5\ M_\odot$ and $P = 0.37$ day, the effects of mixing counteract the effect of the centrifugal support in the late stage of the MS. Thus the luminosity and effective temperature of the rotating model are approximate to those of non-rotating one at the age of 1.5 Gyr. But before the age of about 1.2 Gyr, the rotating model has an obviously lower effective temperature compared to the non-rotating one.** Our calculations show that all stars with $Z = 0.008$ and **masses between around 1.3 and 2.0** $M_\odot$ have similar characteristics.

We also calculated the evolutionary tracks of rotating models with an initial rotation period of 0.49 day and a low efficiency of mixing ($f_c = 0.03$). Figure 4 shows the evolutionary tracks of models with $M = 1.15$ and 1.5 $M_\odot$. For a comparison, the track of the rotating model without mixing ($f_c = 0$) of elements but with other effects of rotation, such as the effect of the centrifugal acceleration, is also plotted in the panel of the star with $M = 1.5\ M_\odot$. The lower the efficiency of mixing, the lower the effects of the mixing. Thus the rotating models with a small f_c have lower effective temperatures compared to the rotating models with a large f_c.

3. Isochrones and synthesis results

The rotating models of stars with masses between about 1.3 and 2.0 $M_\odot$ have a lower effective temperature than non-rotating ones, which would lead to a spread in color of the intermediate-age massive star clusters. Thus we calculated a grid of evolutionary tracks[1] of rotating and non-rotating models with masses between 1.0 and 3.0 $M_\odot$. The mass interval δM is **between 0.01 and 0.02** $M_\odot$ for stars with masses between 1.15 and 2.0 $M_\odot$ but is about 0.1 $M_\odot$ for others. The metallicity Z of evolutionary models was converted firstly into [Fe/H]. Then the theoretical properties ([Fe/H], $T_{\rm eff}$, $\log g$, $\log L$) have been transformed into colors and magnitudes using the color transformation tables of Lejeune et al. (1998).

Figure 5 shows the CMDs of different isochrones obtained from our evolutionary models. **The all rotating models have the same initial rotation period of 0.49 day.** The MSTO of rotating models with age = 1.2 and 1.5 Gyr is almost coincident with that of non-rotating models with age = **1.35** and 1.7 Gyr, respectively. This indicates that the rotationally induced spread in color **is equivalent to the effect of an age spread of about 200 Myr. With the increase or decrease in age, the extension becomes narrower and narrower. For example, when the age increases to 1.7 or decreases to 0.9 Gyr, the extension is equivalent to the effect of an age spread of about 100 Myr. This is because the effect of the centrifugal acceleration is partly counteracted by the effects of mixing. When the age is located between 0.8 and 1.9 Gyr,** rotating models exhibit a redder and fainter turnoff with respect to their non-rotating counterparts, **which leads to the fact that rotating population are apparently 100-200 Myr older than non-rotating counterparts in the CMDs. However, they have the same age.** The evolutions of rotating models with masses between about 1.3 and 2.0 $M_\odot$ are faster than those of non-rotating ones. Thus the mass of the turnoff stars of **the isochrone of** rotating models is slightly less than that of non-rotating ones. Therefore, the MSTO of rotating models is fainter than that of non-rotating ones.

When the age of star cluster is older than 2.4 Gyr, the rotating models exhibit a bluer and brighter MSTO with respect to their non-rotating counterparts; **the rotationally induced spread is equivalent to the effect of an age spread of about 200-400 Myr. When the age of star clusters is younger than 0.6 Gyr, the rotating models also exhibit a bluer and brighter MSTO with respect to their non-rotating counterparts. For example, in a star cluster with an age of 0.4 Gyr, the spread caused by the rotation is similar to the effect of an age spread of 50 Myr. Bastian & Silva-Villa (2013) found that the age spread is less than 35 Myr in NGC 1856 and NGC 1866. When the mass of stars is larger than about 2**

[1]The tracks can be obtained by e-mail to Wuming Yang.

$M_\odot$ **or less than about 1.3** $M_\odot$**, the rotating models mainly exhibit a higher effective temperature than non-rotating ones at the same age. Thus the rotating models have a bluer and brighter turnoff compared to their non-rotating counterparts when age is younger than 0.6 Gyr or older than 2.4 Gyr.**

In order to understand whether rotation can produce the extended or double MSTOs or not, we performed a stellar population synthesis by Monte Carlo simulation, for a total mass of about 3×10^4 $M_\odot$ **(the mass range of the intermediate-age massive star clusters is between about (1-20)**$\times 10^4$ $M_\odot$ **(Goudfrooij et al. 2011b))** following the log-normal initial mass function of Chabrier (2001). **In order to match the result of Milone et al. (2009) that about 70% of stars belong to the blue and bright MSTO and around 30% to the red and faint MSTO,** we simply assumed that 70% of stars do not rotate but 30% of stars rotate with the same initial period of 0.49 day. **This assumption may do not represent the realistic situation.** In the synthesized population, we include observational errors, taken to be a Gaussian distribution with a standard deviation of 0.01 and **0.015 in magnitude and color as that of Bastian & de Mink (2009), respectively. The deviation of 0.015 in V-I corresponds to a deviation of about 90 K in the effective temperature in our models.**

The CMDs of the synthesized star clusters with different ages are shown in Fig. 6. The double MSTOs can be clearly seen in the star clusters with age = 1.2 and 1.5 Gyr, in which the hot and bright MSTO is dominant. The continuously extended MSTO exists in the star cluster with age = 0.9 Gyr, where the hot and bright MSTO is also dominant. Compared to the extension of the MSTO of star clusters with **ages between 0.9 and 1.6 Gyr, that of the star clusters with age = 0.4** and 1.7 Gyr is not obvious. Moreover there is almost no extension of the MSTO of the star cluster with age = 2.0 Gyr. For the star clusters with age = 2.6 and 3.0 Gyr, their MSTO is slightly extended by the effects of rotation, but the cool and faint MSTO is dominant. **The difference in the V-I between the isochrone of rotating models and that of non-rotating ones is less than 0.01 for the star cluster with an age of 0.4 Gyr. Thus the extension of the MSTO in this cluster is dominated by the deviation of 0.015 in color. However, the difference is about 0.04 for the star cluster with an age of 1.2 Gyr. The spreads in star clusters with ages between 1.2 and 1.5 Gyr are mainly due to rotation.**

The isochrones of rotating models with an initial period of about 0.73 day and the synthesized results are shown in Figs. 7 and 8, respectively. For the star clusters with age less than 0.7 Gyr, their MSTOs are not extended by rotating stars. When the age of star clusters is located between 0.9 and 2.2 Gyr, the rotating models have a redder and fainter MSTO compared to the non-rotating counterparts, which is equivalent to the effect of an age spread of about 100-200 Myr. Compared to the results obtained from the rotating models with the initial rotation period of 0.49 day, the results for the star clusters with age larger than 2.6 Gyr are the same; but the red and faint MSTO appears in older intermediate-age star clusters.

Figures 9 and 10 show the results obtained from the rotating models with an initial period of 0.37 day. The interesting scenarios are that the spread caused by rotation disappears in star clusters with ages between 1.4 and 2.0 Gyr, but the spread that is similar to the effect of an age spread of about 100-200 Myr still exists in the star clusters with ages between 0.8 and 1.3 Gyr. When the age is less than 0.7 Gyr, the rotating models have a bluer and brighter MSTO compared to non-rotating ones, which is equivalent to the effect of an age spread of about 100 Myr. This is because the effects of mixing are easier to play a dominant role in the late stage of the MS of stars for a high rotation rate. These results are similar to those of Girardi et al. (2011) except for star clusters with ages between 0.9 and 1.2 Gyr. The MSTO of star clusters with age larger than 2.6 Gyr is similar to that of the star clusters with a lower rotation rate. The efficiency of mixing in our models should be less than that of Girardi et al. (2011). Thus, in our calculations, it needs a

higher rotation rate to produce the similar results.

The NGC 1751 has the double MSTO and its age is about 1300-1500 Myr (Milone et al. 2009). In a star cluster, the initial rotation period of stars should be different. For simplicity, we assumed that its population is composed of 70% non-rotating stars and 30 % rotating stars which is obtained by interpolating between our rotating models with different rotation rates, assuming the initial rotation rate has a Gaussian distribution with a peak at about 0.3 times Keplerian rotation rate on ZAMS and a standard deviation of 0.06. This assumption may do not represent the realistic distribution. For example, the distribution of rotation rate of A0-A1 type stars are more flat (Royer et al. 2007). A distance modulus of 18.5 is adopted in this simulation. The simulated results are shown in Fig. 11, where the double MSTO can be seen. The structure of the simulated MSTO is very similar to the observed by Milone et al. (2009) (see their Fig. 9).

Comparing the isochrones obtained from rotating models with different rotation rates, one can find that the largest extension is almost the same, i.e., can be equivalent to the effect of an age spread of about 200 Myr. With the increase in rotation rate, for the intermediate-age star clusters, the extension of the MSTO will disappear in the star clusters with an older age due to the fact that the effects of mixing counteract the effect of the centrifugal support; for the old-age star clusters, the blue and bright extension of the MSTO will be not affected because magnetic braking dominates the rotation of stars with mass less than about 1.3 $M_{\odot}$; for young-age star clusters, an blue and bright extension of the MSTO can be produced by high rotation.

The observed extension of the MSTO of the intermediate-age star clusters can be equivalent to the effect of an age spread of about 100-500 Myr, which is broader than the extension caused by rotating models with $f_c = 0.2$. Figure 4 shows that the effects of rotation on the evolution of stars are sensitive to the value of f_c. Thus we calculated the rotating population with the initial period of 0.49 day but with a lower efficiency of mixing ($f_c = 0.03$). The results for this calculation are shown in Figs. 12 and 13. When the age of star clusters is located between 1.4 and 1.7 Gyr, the extension of MSTO caused by rotation is equivalent to the effect of an age spread of about 400 Myr, which decreases with increasing or decreasing age. But the CMD of star clusters with age less than 0.6 Gyr is almost not affected by rotation. When the age is larger than 2.6 Gyr, rotating models exhibit a bluer and brighter MSTO compared to non-rotating counterparts, which can be equivalent to the effect of an age spread of about 350 Myr in the cluster with an age of 3.0 Gyr.

4. Discussion and Summary

According to the studies of Wolff et al. (2004), Zorec & Royer (2012), and Yang et al. (2013b), the ZAMS models of A-type stars should be a Wolff ZAMS model which is required for the studies of the evolution of surface velocity and the transport of angular momentum (Yang et al. 2013b). For simplicity in the computation, we adopted the ZAMS model with a uniform rotation. The rotating models computed from the Wolff ZAMS model also exhibit a lower effective temperature and a higher luminosity with respect to non-rotating ones in the late stage of the MS of stars with masses between 1.3 and 2.0 $M_{\odot}$, which is consistent with the results obtained from the evolution of the ZAMS model with a uniform rotation (see Fig. 4). Thus our results about the effects of rotation on the CMDs cannot be **significantly** changed by the ZAMS model.

For a given rotation rate, the coefficient of the diffusion increases with increasing mass, i.e. the efficiency of the mixing increases with increasing mass, **which leads to the fact that there is a critical mass M_c for the effects of rotation on the structure and evolution of stars. For stars with mass larger than M_c,** the effect of the mixing soon excceds the **centrifugal** effect in the early stage of the MS. **For stars with masses between about 1.3 $M_{\odot}$ and M_c, the effect of**

the centrifugal acceleration is dominant for a long time in the early stage of the MS; the more massive the star, the larger the fractional lifetime of the MS that is dominated by **the effect of the centrifugal acceleration** because the **centrifugal** effect also increase with increasing mass for a given rotation rate. **Our ZAMS models with mass larger than 2.0 $M_{\odot}$ have had a little gradient of elements in interior compared to lower mass stars in which the gradient can be neglected. In addition, the MS lifetime is longer for the lower mass stars. Thus the diffusion of elements takes place at the beginning of evolution for stars with mass larger than 2.0 $M_{\odot}$, but it acts in a later time for stars with mass less than 2.0 $M_{\odot}$. Thus the early evolution of the MS of stars with mass less than 2.0 $M_{\odot}$ is easier dominated by the effect of centrifugal acceleration. The critical mass is about 2.0 $M_{\odot}$ in our models, which is close to the critical mass of the He-flash of Yang et al. (2012). It should be affected by the mixing processes. Thus the value of about 2.0 $M_{\odot}$ is debatable.**

In rotating stars, meridional circulation is an advection process. The rotational mixing would be much more complex than that described by a diffusion process. The value of 0.2 of f_c in our models is obviously higher than that calibrated against the nitrogen abundance in massive stars (0.0228) (Hunter et al. 2008; Brott et al. 2011) or against the solar model (0.03) (Yang & Bi 2006). There may be other mixing mechanisms undescribed by present model in stars and Sun (Hunter et al. 2008; Basu & Antia 2008), such as gravity waves which are efficient at transporting angular momentum (Zahn et al. 1997; Talon & Charbonnel 2005). The value of 0.2 for the f_c corresponds to a high efficient mixing in stars, which results in a higher mean density for stars with mass less than 2.0 $M_{\odot}$ when they approach the end of MS, i.e., the models with a high efficient mixing have a lower radius, which aids in explaining the evolution of surface velocity of stars with mass less 2.0 $M_{\odot}$ (Zorec & Royer 2012; Yang et al. 2013b) besides producing the extension of the MSTO that is equivalent to the effect of an age spread of 200 Myr. However, if the mixing is too efficient, which will bring an amount of hydrogen into the burning region of stars. As a consequence, the extension of the MSTO caused by rotation would disappear. Our current understanding of the mixing processes should be limited. And the evolutions of rotating stars should exceed the predictive power of one-dimensional rotating stellar models. The mixing in stars is debatable.

The rotationally induced extension of MSTO do not exist in the simulated clusters with age between 1.4 and 2.0 Gyr when the initial rotation period is 0.37 day, which is similar to the results of Girardi et al. (2011). In addition, the extension also disappears in the young clusters with the rotating models with $P_0 = 0.73$ day. If we took a fixed rotation velocity V_0 as the initial rotation parameter for all models, the high mass stars would have a lower rotation rate but the low mass stars would have a higher rotation rate. As a consequence, one could obtain the result that rotation cannot affect obviously the MSTO of star clusters. Moreover, the efficiency of mixing should be different between our models and Girardi et al. (2011) models because there is no the adjustable parameter f_c in the Geneva stellar evolution code (Maeder & Meynet 2000). The different initial rotation parameter and efficiency of mixing would lead to the differences between our results and Girardi et al. (2011)'s.

Compared with Bastian & de Mink (2009) models, we calculated a dense grid of evolutionary tracks of rotating models and considered the effects of rotation on the lifetime of the MS in our isochrones and simulations. Our results are similar to that of Bastian & de Mink (2009), i.e., the MSTO of star clusters with ages between about 0.8 and 2.2 Gyr can be extended by stellar rotation. The extension is dependent on rotation rate, the efficiency of mixing, and the age of star clusters.

For stars with masses between about 1.3 and 2.0 $M_{\odot}$, the effect of the centrifugal support plays a dominant role in the early stage of evolutions, which leads to the fact that rotating models have a lower effective temperature and evolve faster than non-rotating ones. As the evolutions proceed, the effect of rotational mixing partly counteracts the influence of the centrifugal acceleration, even dominates the effects of rotation on the structure and evolution of stars. As a consequence, the MSTO of intermediate-age star clusters with ages between 0.8 and 2.2 Gyr can be extended redward, but the extension decreases with increasing or decreasing age. In the rotating stars with mass less than 1.3 $M_{\odot}$, the effect of rotational mixing plays a dominant role during the MS stage, which results in the fact that rotating models have a higher effective temperature than non-rotating ones at the same age. Thus the MSTO of the older star clusters is extended blueward. The evolutions of rotating stars with mass larger than 2.0 $M_{\odot}$ are mainly affected by the effect of rotational mixing which depends on rotation rate and the efficiency of mixing. Therefore, the MSTO of young star clusters can be slightly extended blueward or cannot be extended, which is dependent on the rotation rate and the efficiency of the mixing.

In the 16 star clusters studied by Milone et al. (2009) (see their Table 3), the star clusters with ages between 950 and 1400 Myr have an extended MSTO of about 150-250 Myr, but the star clusters with age larger than 1550 Myr have a smaller extension (Δage < 100 Myr), which seems to be similar to our results that rotationally induced extension decreases with the increase or decrease in the age of star clusters and to be more similar to the results obtained from the high-efficiency models than to those obtained from the low-efficiency models.

The extension of the MSTO caused by rotation is significantly affected by the efficiency of mixing and decreases with increasing the efficiency. The star cluster SL 529 with an age of about 2.0 Gyr has an age spread of about 500 Myr (Piatti 2013). The rotationally induced extension of the MSTO of star cluster with an age of 2.0 Gyr is similar to the effect of an age spread of about 250 Myr in our models with $f_c = 0.03$ (see Fig. 12). If we took a lower initial rotation rate, the spread that can be equivalent to the effect of an age spread of 400 Myr would appear in star clusters older than 1.7 Gyr. Thus the spread of star cluster SL 529 seems to be not conflicting with our results.

In this work, we calculated a grid of evolutionary tracks of rotating **and non-rotating** stars with $Z = 0.008$ and masses between 1.0 and 3.0 $M_{\odot}$. For a given rotation rate, the effects of rotation on the structure and evolution of stars are dependent on the mass and age of stars. **For the initial rotation period of 0.49 day and the efficiency of mixing of 0.2, rotating models have a fainter and redder turnoff with respect to the non-rotating counterparts when age is located between about 0.8 and 1.9 Gyr. When the age is larger than 2.4 Gyr, rotating models have a brighter and bluer turnoff with respect to the non-rotating ones. However, when the age is less than 0.6 Gyr, the rotation almost not affects the MSTO of star clusters. The extension of the MSTO caused by the rotation can be equivalent to an age spread of about 200 Myr when the age is located between about 1.2 and 1.6 Gyr. The extension decreases with the decrease or increase in age. When the initial rotation period decreases to about 0.37 day, the extension of the MSTO caused by rotation disappears in star clustars with ages between about 1.4 and 2.0 Gyr, young star clusters with age less than 0.7 Gyr have a blueward extended MSTO; but star clusters with ages between 0.8 and 1.3 Gyr still have a redward extended MSTO. When the initial rotation period increases to about 0.73 day, the MSTO of star clusters with age less than 0.7 Gyr cannot be affected by rotation, but the MSTO of star clusters with ages between 0.9 and 2.2 Gyr is extended redward. When the value of the efficiency of mixing decreases to 0.03, the extension of the MSTO caused by rotation can be equiv-**

alent to the effect of an age spread of about 400 Myr. The rotationally induced extension of the MSTO of intermediate-age star clusters is dependent on the rotation rate, the age of star clusters, and the efficiency of the mixing of elements. But the blueward extension of the MSTO of star clusters with age larger than about 2.4 Gyr is mainly dependent on the efficiency of the mixing.

Our simulation shows that rotation could lead to the double or extended MSTO in the star clusters with ages between 0.8 and 2.2 Gyr or larger than 2.4 Gyr. The rotation seems to cannot result in the extension of the MSTO of star clusters with age less than 0.6 Gyr; at least, the extension of the MSTO of these star clusters is not significant, compared to that of intermediate-age star clusters. Considered the difference in the initial rotation rates and the efficiency of mixing, the redward extended MSTO may be easier to be observed in star clusters with ages between about 0.9 and 1.5 Gyr.

We **thank the anonymous referee for helpful comments and** acknowledge the support from the NSFC 11273012, 11273007, 10933002, and 11003003, and the Project of Science and Technology from the Ministry of Education (211102).

REFERENCES

Bastian, N., & de Mink, S. E. 2009, MNRAS, 398, L11

Bastian, N., & Silva-Villa, E. 2013, MNRAS, eprint arXiv:1302.3376

Basu, Sarbani, Antia, H. M. 2008, PhR, 457, 217

Bi, S. L., Li, T. D., Li, L. H., & Yang, W. M. 2011, ApJ, 731, L42

Brocato, E., Castellani, V., Di Carlo, E., Raimondo, G., & Walker, A. R. 2003, AJ, 125, 3111

Brott, I. et al. 2011, A&A, 530, A116

Chaboyer, B., & Zahn, J.-P. 1992, A&A, 253, 173

Chabrier, G. 2001, ApJ, 554, 1274

D'Antona, F., Caloi, V., Montalb*á*n, J., Ventura, P., & Gratton, R. 2002, A&A, 395, 69

Decressin, T., Meynet, G., Charbonnel, C., Prantzos, N., & Ekström, S. 2007, A&A, 464, 1029

De Mink, S. E., Pols, O. R., Langer, N., & Izzard, R. G. 2009, A&A, 507, L1

Eggenberger, P., Miglio, A., Montalban, J., Moreira, O., Noels, A., Meynet, G., & Maeder, A. 2010, A&A, 509, A72

Endal, A. S., & Sofia, S. 1978, ApJ, 220, 279

Georgy, C., Ekström, S., Granada, A. et al. 2013, A&A, eprint: 1303.2321

Girardi, L., Rubele, S., & Kerber, L. 2009, MNRAS, 394, L74

Girardi, L., Eggenberger, P., & Miglio, A. 2011, MNRAS, 412, L103

Girardi, L. et al. 2013, MNRAS, tmp.1085

Glatt, K. et al. 2008, AJ, 136, 1703

Goudfrooij, P., Puzia, T. H., Kozhurina-Platais, V., & Chandar, R. 2009, AJ, 137, 4988

Goudfrooij, P., Puzia, T. H., Kozhurina-Platais, V., & Chandar, R. 2011, ApJ, 737, 3

Goudfrooij, P., Puzia, T. H., Chandar, R., & Kozhurina-Platais, V. 2011, ApJ, 737, 4

Gratton, R., Sneden, C., & Carretta, E. 2004, ARA&A, 42, 385

Hunter, I. et al. 2008, ApJ, 676, L29

Julien, K. 1996, Dyn. Atmosph. Oceans, 24, 237

Keller, S. C., Mackey, A. D., & Da Costa, G. S. 2011, ApJ,

Keller, S. C., Mackey, A. D., & Da Costa, G. S. 2012, ApJ,

Kerber, L. O., Santiago, B. X., Brocato, E. 2007, A&A, 462, 139

Lejeune, T., Cuisinier, F., & Buser, R., 1998, A&A, 130, 65

Mackey, A. D., & Broby Nielsen, P. 2007, MNRAS, 379, 151

Mackey, A. D., Broby Nielsen, P., Ferguson, A. M. N., & Richardson, J. C. 2008, ApJ, 681, L17

Maeder, A. 1987, A&A, 178, 159

Maeder, A., & Meynet, G. 2000, ARA&A, 38, 143

Milone, A. P., Bedin, L. R., Piotto, G., & Anderson, J. 2009, A&A, 497, 755

Milone, A. P. et al. 2010, ApJ, 709, 1183

Milone, A. P. et al. 2013, ApJ, 767, 120

Piatti, A. E. 2013, MNRAS, eprint arXiv:1301.1627

Pinsonneault, M. H., Kawaler, S. D., Sofia, S., & Demarqure, P. 1989, ApJ, 338, 424

Piotto, G. et al. 2005, ApJ, 621, 777

Piotto, G. et al. 2007, ApJ, 661, L53

Platais I. et al. 2012, ApJ, 751, L8

Royer, F., Zorec, J., & Gómez, A. E. 2007, A&A, 463, 671

Rubele, S., Kerber, L., & Girardi, L. 2010, MNRAS, 403, 1156

Rubele, S., Girardi, L., Kozhurina-Platais, V., Goudfrooij, P., & Kerber, L. 2011, MNRAS, 414, 2204

Talon S., & Charbonnel C. 2005, A&A, 440, 981

Talon, S., Zahn, J.-P., Maeder, A., & Meynet, G. 1997, A&A, 322, 209

Ventura, P., D'Antona, F., Mazzitelli, I., & Gratton, R. 2001, ApJ, 550, L65

Wolff, S. C., Strom, S. E., & Hillenbrand, L. A. 2004, ApJ, 601, 979

Yang, W. M., & Bi, S. L. 2006, A&A, 449, 1161

Yang, W. M., & Bi, S. L. 2007, ApJ, 658, L67

Yang, W. M., Meng, X. C., Bi, S. L., Tian, Z., Li, T., & Liu, K. 2011, ApJ, 731, L37

Yang, W. M., Meng, X. C., Bi, S. L., Tian, Z., Liu, K., Li, T., & Li, Z. 2012, MNRAS, 422, 1552

Yang, W., Bi, S., & Meng, X. 2013a, RAA, 13, 579

Yang, W., Bi, S., Meng, X., & Tian, Z. 2013b, ApJ, 765, L36

Zahn J.-P., Talon S., & Matias J. 1997, A&A, 322, 320

Zorec, J., & Royer, F. 2012, A&A, 537, A120

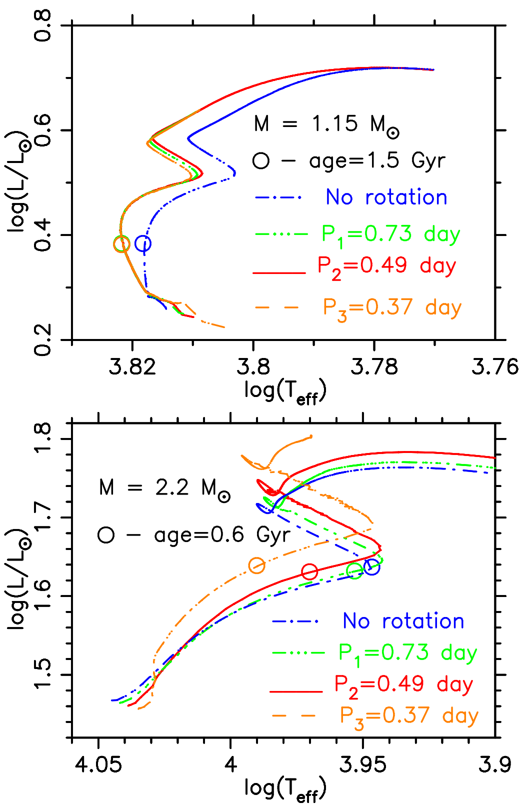

Fig. 1.— Evolutionary tracks of rotating and non-rotating models in the Hertzsprung-Russell (HR) diagram. The value of $P_i (i = 1, 2, 3)$ is the initial period on the ZAMS. The notation '$\circ$' shows a position of the models with a given age.

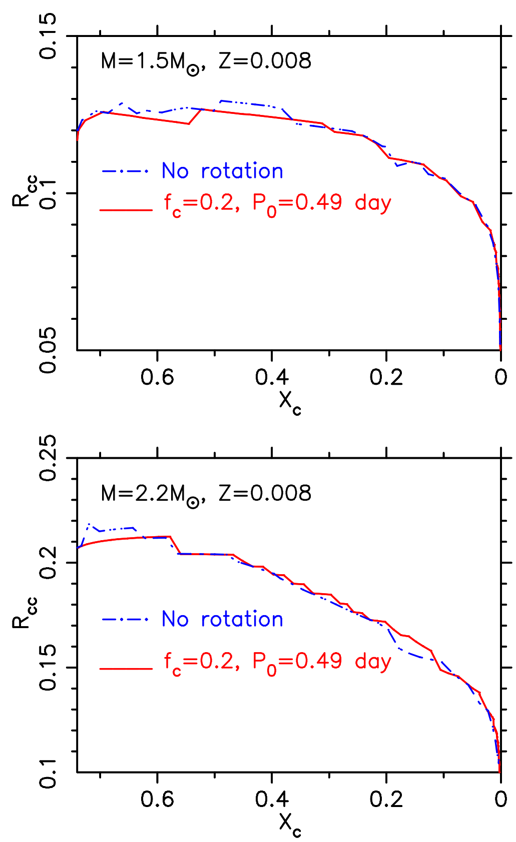

Fig. 2.— The radius of the convective core of stars as a function of the central hydrogen abundance.

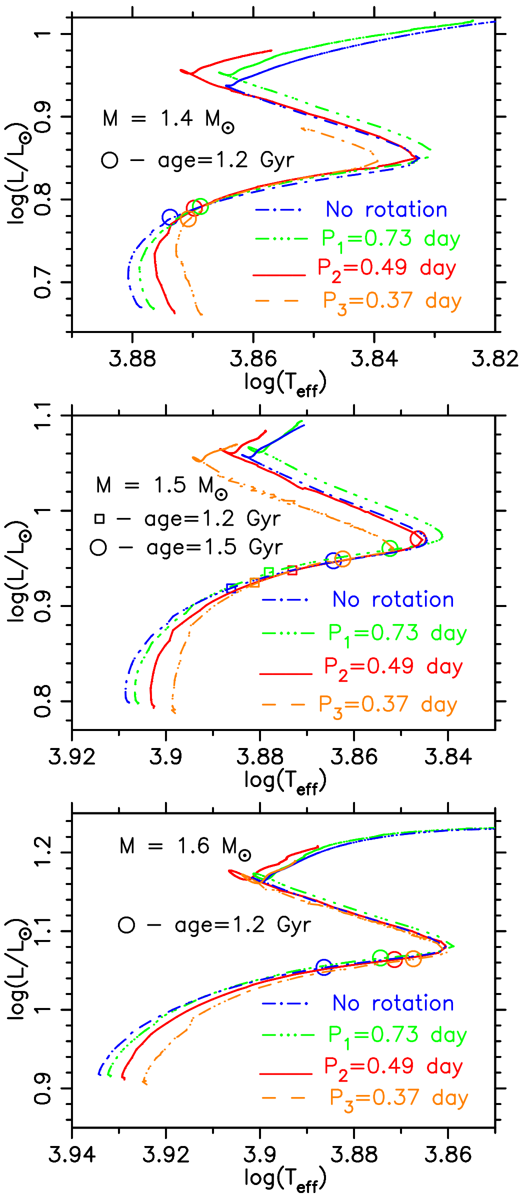

Fig. 3.— Same as Fig. 1 but for models with different masses.

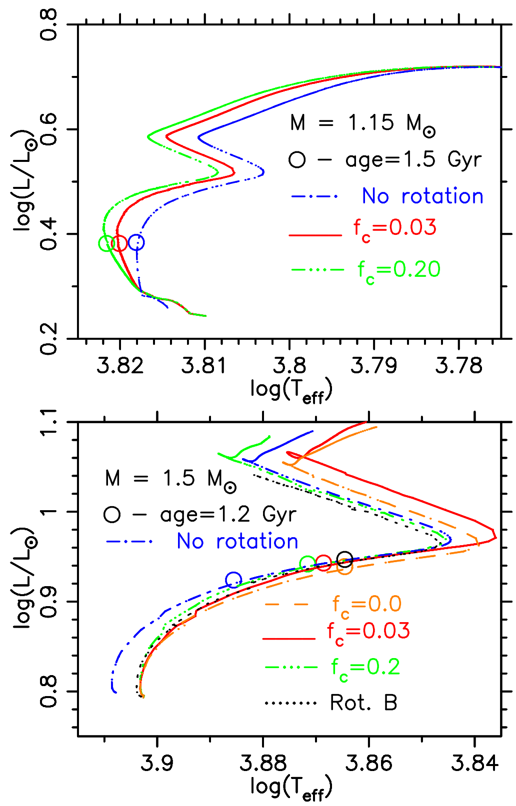

Fig. 4.— Same as Fig. 1 but rotating models with the same initial period (0.49 day) but with different efficiencies of element mixing (f_c). The evolution of Rot. B with $f_c = 0.2$ was computed from the Wolff ZAMS (Yang et al. 2013b) to the end of the MS.

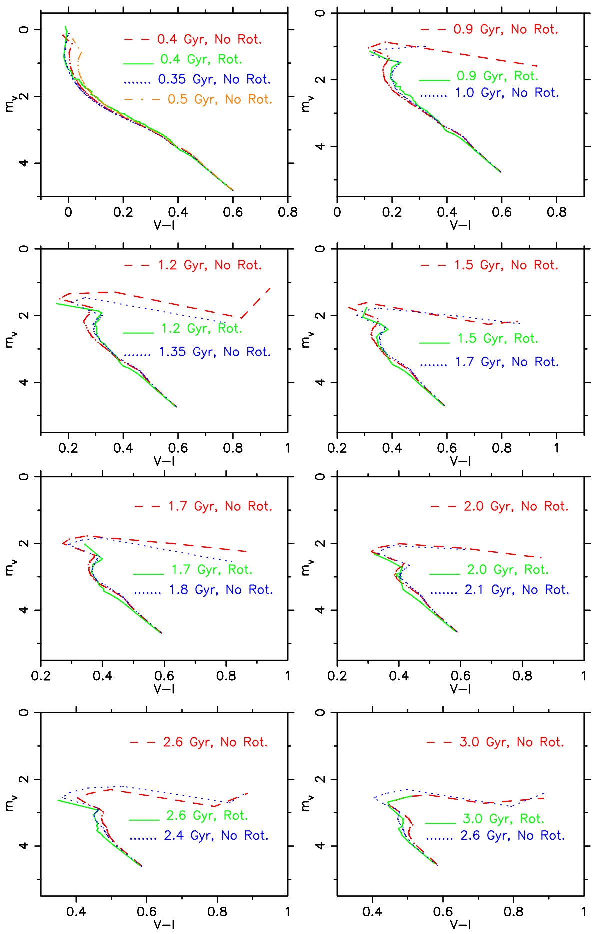

Fig. 5.— The CMDs of the different isochrones. The solid (green) lines show the isochrones of rotating models with an initial period of about 0.49 day and $f_c = 0.2$, while the dashed (red) and dotted (blue) lines indicate the isochrones of non-rotating models with different ages.

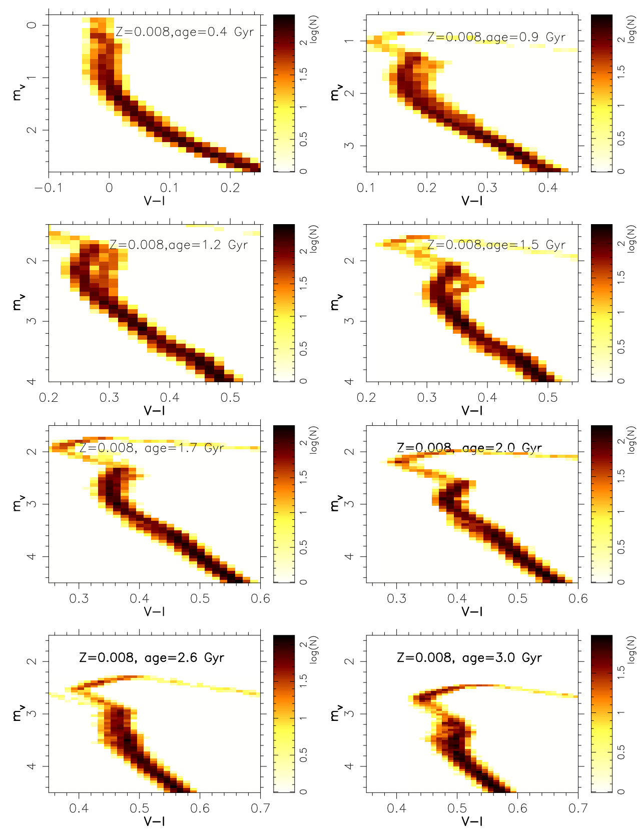

Fig. 6.— The CMDs of the synthetic star clusters at different ages. The total mass is assumed to be of about 3×10^4 $M_{\odot}$, following a Chabrier (2001) log-normal initial mass function. We assumed 30% of stars rotating with $P_0 = 0.49$ day and $f_c = 0.2$.

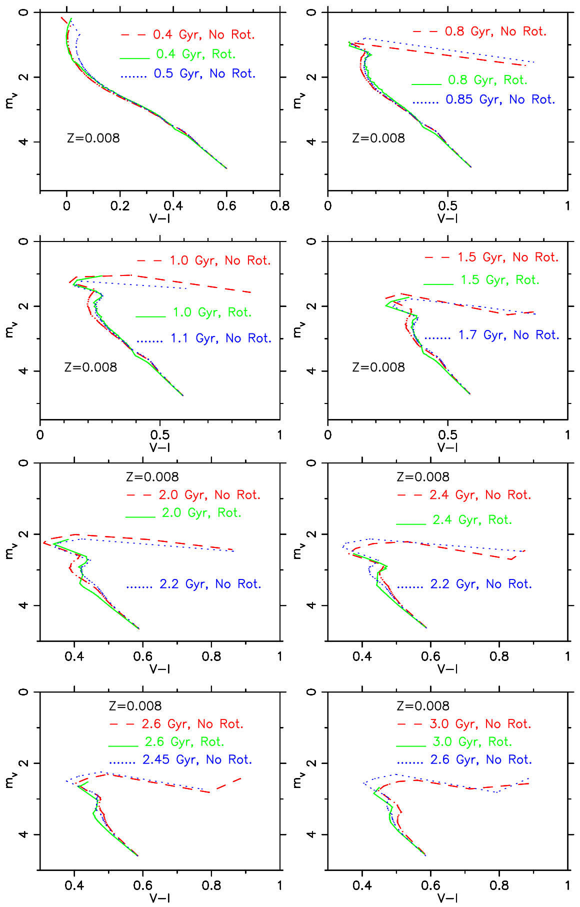

Fig. 7.— Same as Fig. 5 but the initial period of rotating models is about 0.73 day.

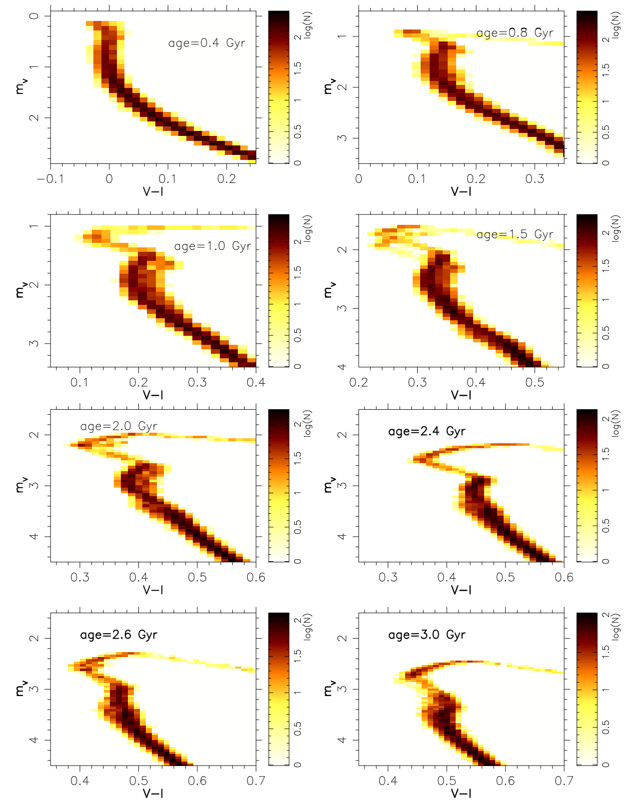

Fig. 8.— Same as Fig. 6 but the initial period of rotating population is 0.73 day.

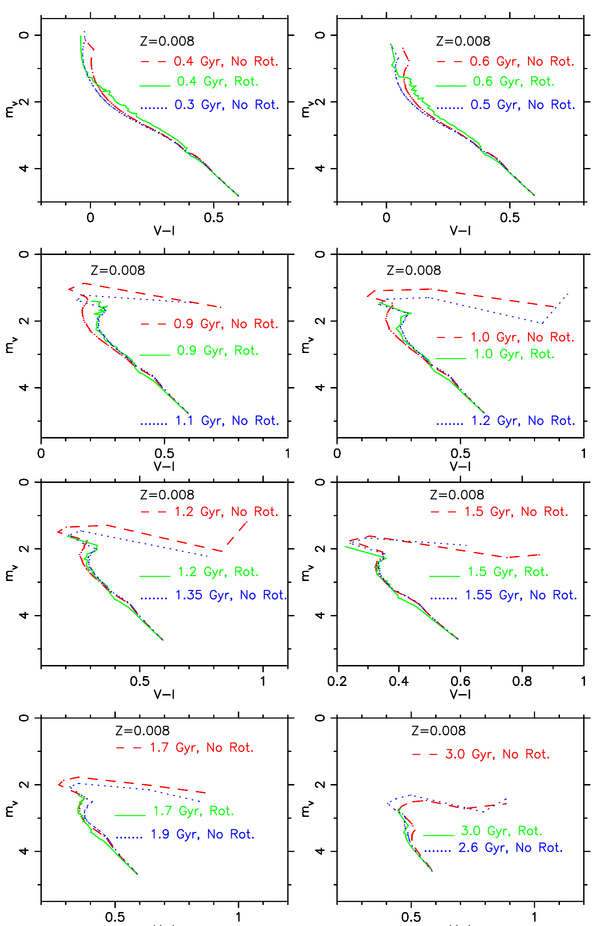

Fig. 9.— Same as Fig. 5 but the initial period of rotating models is about 0.37 day.

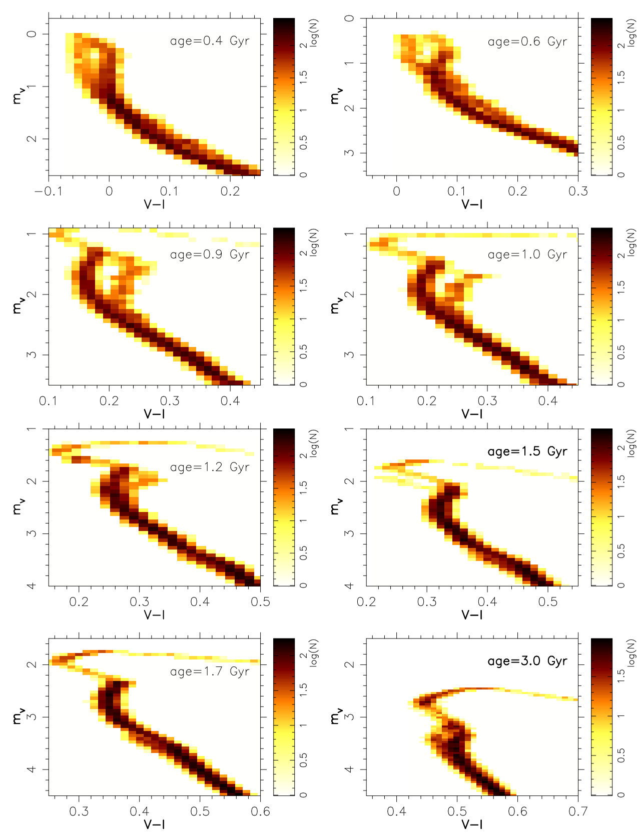

Fig. 10.— Same as Fig. 6 but the initial period of rotating population is 0.37 day.

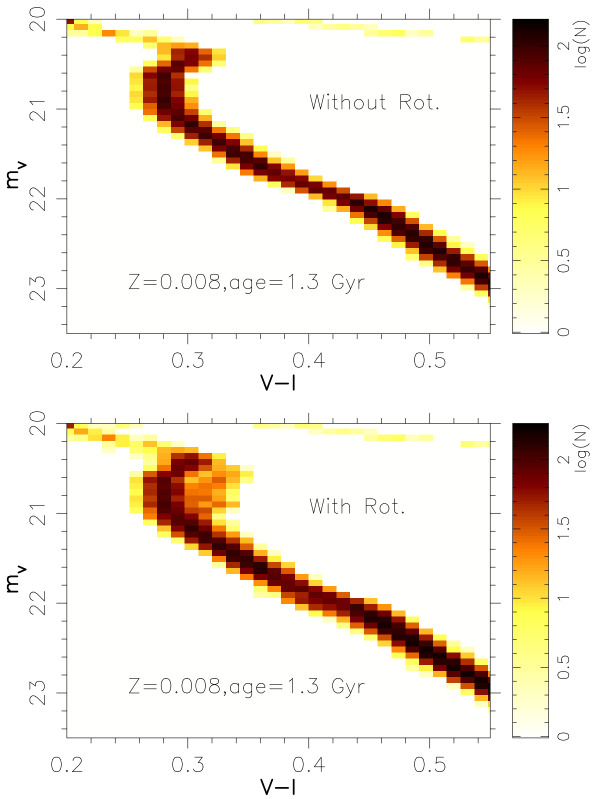

Fig. 11.— The CMDs of a simulated star cluster with the age of NGC 1751. A distance modulus of 18.5 is adopted. Top panel shows the cluster without rotating population; bottom panel represents the cluster with 30% rotating population obtained by interpolating between rotating models with different rotation rate.

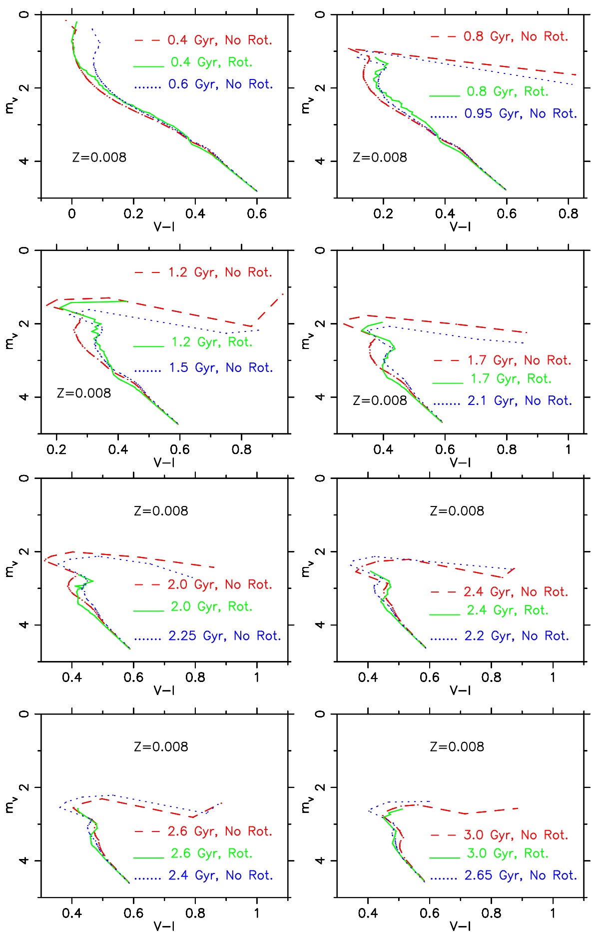

Fig. 12.— Same as Fig. 5 but the value of f_c is 0.03.

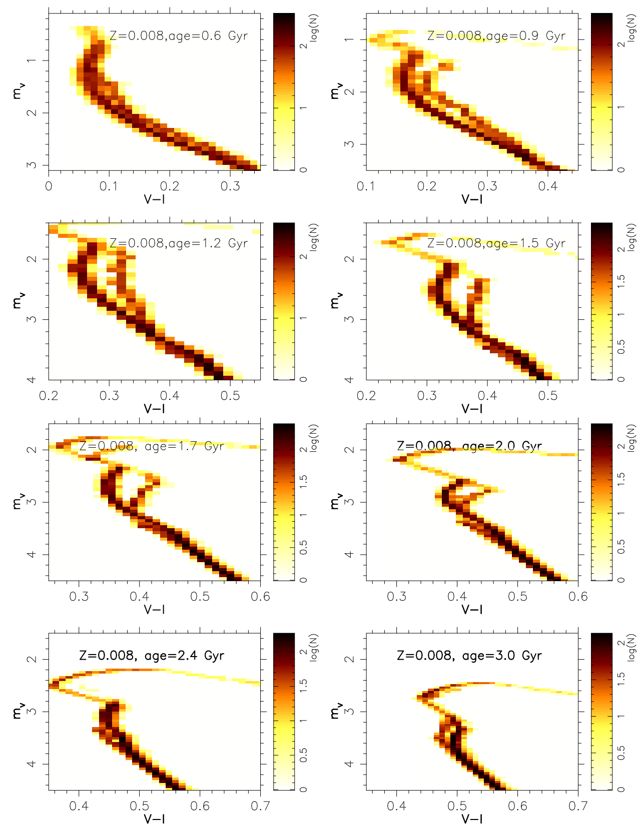

Fig. 13.— Same as Fig. 6 but the value of f_c is 0.03.